JEE Mains 2020

9th Jan Shift 2 Paper – Mathematics

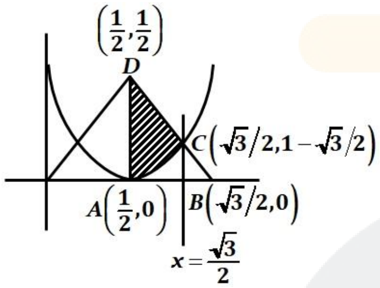
1. If $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ 1/2 & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$, then find the area bounded by $f(x)$ and $g(x) x = \frac{1}{2}$ to $x = \frac{\sqrt{3}}{2}$

A) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ B) $\frac{\sqrt{3}}{4} + \frac{1}{3}$ C) $2\sqrt{3}$ D) $3\sqrt{3}$

Ans. A

Sol. Required area = Area of trapezium $ABCD - \int_{t/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$



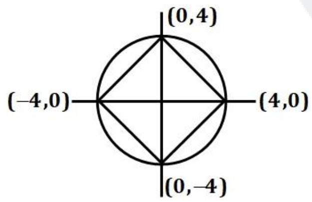
$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\left(x - \frac{1}{2}\right)^3\right)_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{4} - \frac{1}{3}$$

2. z is a complex number such that $|Re(z)| + |\text{lim}(z)| = 4$ then $|z|$ can't be

A) $\sqrt{7}$ B) $\sqrt{10}$ C) $\sqrt{\frac{17}{2}}$ D) $\sqrt{8}$

Ans. A

Sol. $z = x + iy$



$|x| + |y| = 4$

Minimum value of $|z| = 2\sqrt{2}$

Minimum value of $|z| = 4$

$|z| \in [\sqrt{8}, \sqrt{16}]$

So, $|z|$ can't be $\sqrt{7}$

3. If $f(x) = \begin{vmatrix} x+1 & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ and $a - 2b + c = 1$, then

A) $f(50) = 1$ B) $f(-50) = -1$ C) $f(50) = 501$ D) $f(50) - 501$

Ans. A

Sol. Apply $R_1 = R_1 + R_3 - 2R_2$

$$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} \Rightarrow f(x) = 1 \Rightarrow f(50) = 1$$

4. Let a_n is a positive term of a GP and $\sum_{n=1}^{100} a_{2n+1} = 200$, $\sum_{n=1}^{100} a_{2n} = 100$, find $\sum_{n=1}^{200} a_n$.

A) 300 B) 150 C) 175 D) 225

Ans. B

Sol. Let GP is $a, ar, ar^2 \ldots\ldots$

$$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots a_{201} = 200$$

$$\Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200 \qquad (1)$$

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots a_{200} = 100$$

$$\Rightarrow \frac{ar(r^{200}-1)}{r^2-1} = 100 \qquad (2)$$

Form (1) and (2) $r = 2$ add both

$\Rightarrow a_2 + a_3 + \ldots\ldots. a_{200} + a_{201} = 300 \Rightarrow (a_1 + \ldots\ldots. a_{200}) = 300$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

5. If $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, $y(1) = 1$ and $y(x) = e$ then $x = ?$

A) $\frac{\sqrt{3}}{2}e$ B) $\sqrt{3}e$ C) $\sqrt{2}e$ D) $\frac{e}{\sqrt{2}}$

Ans. B

Sol. Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2}$$

$$\Rightarrow \frac{1+v^2}{v^3}dv = -\frac{1}{x}dx$$

$$\Rightarrow \int \left(\frac{1}{v^3} + \frac{1}{v}\right) dv = \int \frac{-1}{x}dx$$

$$\Rightarrow \frac{-1}{2}\frac{1}{v^2} + \ln v = -\ln x + c$$

$$\Rightarrow -\frac{x^2}{2y^2} = -\ln y + c$$

When $x = 1, y = 1$, then

$$-\frac{1}{2} = c$$

$$\Rightarrow x^2 = y^2(1 + 2\ln y)$$

$$\Rightarrow x^2 = e^2(3) \Rightarrow x = \pm\sqrt{3}e$$

So, $x = \sqrt{3}e$

6. Let Probability distribution is $\begin{matrix} X_1: & 1 & 2 & 3 & 4 & 5 \\ P_i: & k^2 & 2k & k & 2k & 5k^2 \end{matrix}$, then value of $P(X > 2)$

A) $\frac{7}{12}$ B) $\frac{1}{36}$ C) $\frac{1}{6}$ D) $\frac{23}{36}$

Ans. D

Sol. $\sum P_i = 1 \Rightarrow 6k^2 + 5k = 1$

$6k^2 + 5k - 1 = 0$

$6k^2 + 6k - k - 1 = 0$

$(6k - 1)(k + 1) = 0$

$\Rightarrow k = -1$ (rejected); $k = \frac{1}{6}$

$P(X > 2) = k + 2k + 5k^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36}$

$= \frac{6 + 12 + 5}{36} = \frac{23}{36}$

7. $\int \frac{d\theta}{\cos^2\theta(\sec 2\theta+\tan 2\theta)} = \lambda\tan\theta + 2\log f(x) + c$, then ordered pair $(\lambda, f(x))$ is-

A) $(1, 1+\tan\theta)$
B) $(1, 1-\tan\theta)$
C) $(-1, 1+\tan\theta)$
D) $(-1, 1-\tan\theta)$

Ans. C

Sol. $\int \frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta}+\frac{2\tan\theta}{1-\tan^2\theta}} dx$

$= \int \frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)\text{^}2} d\theta$

$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta} d\theta$

$\tan\theta = t \Rightarrow \sec^2\theta d\theta = dt$

$= \int \left(\frac{1-t}{1+t}\right) dt = \int \left(-1+\frac{2}{1+t}\right) dt$

$-t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1+\tan\theta) + C$

$\Rightarrow \lambda = -1$ and $f(x) = 1+\tan\theta$

8. If $p \to (p \wedge \sim q)$ is false. Truth value of p & q will be

A) TT
B) TF
C) FT
D) FF

Ans. A

Sol.

p	q	$\sim q$	$p \wedge \sim q$	$p \to (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

9. If $\lim_{x\to 0} d\left[\frac{4}{x}\right] = A$ then the value of x at which $f(x) = [x^2]\sin\pi x$ is discontinuous (where $[\cdot]$ denotes greatest integer function)

A) $\sqrt{A+1}$
B) $\sqrt{A+21}$
C) $\sqrt{A}$
D) $\sqrt{A+5}$

Ans. A

Sol. $\lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$

$\Rightarrow \lim_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = A$

$\Rightarrow 4 - 0 = A$

Check when

A) $x = \sqrt{A+1} \Rightarrow x = \sqrt{5} \Rightarrow$ discontinuous

B) $x = \sqrt{A+21} \Rightarrow x = 5 \Rightarrow$ continuous

C) $x = \sqrt{A} \Rightarrow x = 2 \Rightarrow$ continuous

D) $x = \sqrt{A+5} \Rightarrow x = 3 \Rightarrow$ continuous

10. Let one end of focal chord of parabola $y^2 = 8x$ is $\left(\frac{1}{2}, -2\right)$, then equation of tangent at other end of this focal chord is

A) $x + 2y + 8 = 0$
B) $x + 2y = 8$
C) $x - 2y = 8$
D) $x - 2y + 8 = 0$

Ans. D

Sol. Let $\left(\frac{1}{2}, -2\right)$ is $(2t^2, 4t) \Rightarrow t = \frac{-1}{2}$

Parameter of other end of focal chord is

$2 \Rightarrow$ point is $(8, 8)$

$\Rightarrow$ equation of tangent is

$8y - 4(x+8) = 0 \Rightarrow 2y - x = 8$

11. Let $x+6y=8$ is tangent to standard ellipse where minor axis is $\frac{4}{\sqrt{3}}$, then eccentricity of ellipse is

A) $\sqrt{\frac{5}{6}}$ B) $\sqrt{\frac{11}{12}}$ C) $\frac{1}{3}\sqrt{\frac{11}{3}}$ D) $\frac{1}{4}\sqrt{\frac{11}{12}}$

Ans. B

Sol. $2b=\frac{4}{\sqrt{3}} \Rightarrow b=\frac{2}{\sqrt{3}}$

Equation of tangent $\equiv y=mx\pm\sqrt{a^2m^2+b^2}$

Comparing with $\equiv y=\frac{-x}{6}+\frac{4}{3}\equiv y=\frac{-x}{6}+\frac{4}{3}$

$m=\frac{-1}{6}$ and $a^2m^2+b^2=\frac{16}{9}\Rightarrow\frac{a^2}{36}+\frac{4}{3}=\frac{16}{9}\Rightarrow\frac{a^2}{36}=\frac{16}{9}-\frac{4}{3}=\frac{4}{9}\Rightarrow a^2=16$

$e=\sqrt{1-\frac{b^2}{a^2}}$

$e=\sqrt{1-\frac{4}{3\times 16}}=\sqrt{\frac{11}{12}}$

12. If $f(x)$ and $g(x)$ are continuous functions, fog is identify function, $f'(b)=5$ and $g(b)=a$, then $f'(a)$ is

A) $\frac{1}{5}$ B) $\frac{1}{5}$ C) $\frac{3}{5}$ D) 5

Ans. B

Sol. $f(g))=x\Rightarrow f'(g(x))\cdot g'(x)=1$

Put $x=b\Rightarrow f'(g(b))g'(b)=1\Rightarrow f'(a)\times 5=1\Rightarrow f'(a)=\frac{1}{5}$

$7x+6y-2z=0$

13. If $3x+4y+2z=0$, then which option is correct.

$x-2y-6z=0$

A) No solution B) Only trivial solution

C) Infinite non trivial solution for $x=2z$ D) Infinite non trivial solution for $y=2z$

Ans. C

Sol. $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$

$=7(-20)-6(-20)-2(-10)=-140+120+20=0$

So infinite non-trivial solution exist

Now equation (1)+3 equation (3)

$10x-20z=0$

$x=2z$

14. Let $x=2\sin\theta-\sin 2\theta$ and

$y=2\cos\theta-\cos 2\theta$

Find $\frac{d^2y}{dx^2}$ at $\theta=\pi$

E) $\frac{3}{8}$ F) $\frac{3}{2}$ G) $\frac{5}{8}$ H) $\frac{7}{8}$

Ans. A

Sol. $\frac{dx}{d\theta}=2\cos\theta-2\cos 2\theta$

$\frac{dy}{d\theta}=-2\sin\theta+2\sin 2\theta$

$\therefore\frac{dy}{dx}=\frac{\sin 2\theta-\sin\theta}{\cos\theta-\cos 2\theta}=\frac{2\sin\frac{\theta}{2}\cdot\cos\frac{3\theta}{2}}{2\sin\frac{\theta}{2}\sin\frac{3\theta}{2}}=\frac{\cot 3\theta}{2}$

$\frac{d^2y}{dx^2}=\frac{-3}{2}\operatorname{cosec}^2\frac{3\theta}{2}\cdot\frac{d\theta}{dx}$

$\frac{d^2y}{dx^2}=\frac{-\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}}{2(\cos\theta-\cos 2\theta)}$

$\frac{d^2y}{dx^2(\theta=\pi)}=-\frac{3}{4(-1-1)}=\frac{3}{8}$

15. $f(x):[0,5]\to R, f(x)=\int_0^x x^2\,g(x), f(1)=3$

$g(x)=\int_1^x f(t)\,dt$ then correct choice is

A) $f(x)$ has local minimum at $x=1$

B) $f(x)$ has local maximum at $x=1$

C) $f(x)$ has point of inflection at $x=1$

D) $f(x)$ has no critical point

Ans. A

Sol. $f'(x)=x^2g(x)$

$\Rightarrow f'(1)=1\cdot g(1)=0$ (i) $(\because g(1)=0)$

Now $f''(x)=2xg(x)+x^2g'(x)$

$\Rightarrow f''(x)=2xg(x)+x^2f(x)$ $(\because g'(x)=f(x))$

$\Rightarrow f''(1)=0+1\times$

$\Rightarrow F''(1)=3$ (ii)

From (i) and (ii) $f(x)$ has local minimum at $x=1$

16. Let both root of equation $ax^2-2bx+5=0$ are α and root of equation $x^2-2bx-10=0$ are α and β. Find the value of $\alpha^2+\beta^2$

A) 20

B) 25

C) 15

D) 30

Ans. B

Sol. $2\alpha=\frac{2b}{a}\Rightarrow\frac{\alpha b}{a}$ and $\alpha^2=\frac{5}{a}\Rightarrow\frac{b^2}{a^2}=\frac{5}{a}$

$\Rightarrow b^2=5a$ (i) $(a\neq 0)$

$\alpha+\beta=2b$ (ii)

and $\alpha\beta=-10$ (iii)

$\alpha=\frac{b}{a}$ is also root of $x^2-2bx-10=0$

$\Rightarrow b^2-2ba^2-10a^2=0$

by (i) $\Rightarrow 5a-10a^2-10a^2=0\Rightarrow 20a^2=5a\Rightarrow a=\dfrac{1}{4}$ and $b^2=\dfrac{5}{4}$

$\alpha^2=20\ and\ \beta^2=5$

Now $\alpha^2+\beta^2=5+20=25$

17. Let $A=\{x:|x|<2\}$ and $B=\{x:|x-2|\geq 3\}$ then

A) $A\cap B=[-2,-1]$

B) $A\cup B=R-(2,5)$

C) $A-B=[-1,2)$

D) $B-A=R-(-2,5)$

Ans. D

Sol. $A=\{x:x\in(-2,2)\}$

$B=\{x:x\in(-\infty,-1]\cup[5,\infty)\}$

$A\cap B=\{x:x\in(-2,-1]\}$

$A\cup B=\{x:x\in(-\infty,2)\cup[5,\infty)\}$

$A-B=\{x:x\in(-1,2)\}$

$B-A=\{x:x\in(-\infty,-2]\cup[5,\infty)\}$

18. Let $x=\sum_{n=0}^{\infty}(-1)^n(\tan\theta)^{2n}$ and $y=\sum_{n=0}^{\infty}(\cos\theta)^{2n}$ where $\theta\in\left(0,\frac{\pi}{4}\right)$, then

A) $x(y+1)=1$

B) $y(1-x)=1$

C) $y(x-1)=1$

D) $y(1+x)=1$

Ans. B

Sol. $y=1+\cos^2\theta+\cos^4\theta+\cdots\Rightarrow y=\frac{1}{1-\cos^2\theta}\Rightarrow\frac{1}{y}=\sin^2\theta$

$x=\dfrac{1}{1-(-\tan^2\theta)}=\dfrac{1}{\sec^2\theta}\Rightarrow\cos^2\theta=x$

$\Rightarrow\dfrac{1}{y}+x=1\Rightarrow y(1-x)=1$

19. Let the distance between plane passing through lines $\frac{x+1}{2}=\frac{y-3}{2}=\frac{z+1}{8}=8$ and $\frac{x+3}{2}=\frac{y+2}{1}=\frac{z-1}{\lambda}$ and plane $23x-10y-2z+48=0$ is $\frac{k}{\sqrt{633}}$ then k is equal to

A) 1

B) 2

C) 3

D) 4

Ans. C

Sol. Lines must be intersecting

$\Rightarrow(2s-1,3s+3,8s-1)=2t-3,t-2,\lambda t+1)$

$2s-1=2t-3, 3s+3=t-2, 8s-1=\lambda t+1 \Rightarrow t=-1, s=-2, \lambda=18$

Distance of plane contains given lines from given plane is same as distance between point $(-3,-2,1)$ from given plane.

Required distance equal to $\frac{|-69+20-2+48|}{\sqrt{529+100+4}}=\frac{3}{\sqrt{633}}=\frac{k}{\sqrt{633}} \Rightarrow k=3$

20. If ${}^{25}C_0+5\,{}^{25}C_1+9\,{}^{25}C_2 \ldots 101\,{}^{25}C_{25}=2^{25}k$ find $k=?$

Ans. 51

Sol. $\sum_{r=0}^{25}(4r+1)\,{}^{25}C_r=4\sum_{r=0}^{25} r\cdot{}^{25}C_r+\sum_{r=0}^{25}{}^{25}C_r$

$$=4\sum_{r=a}^{25} r\times\frac{25}{r}\,{}^{24}C_{r-1}+2^{25}=100\sum_{r=1}^{25}{}^{24}C_{r-1}+2^{25}$$

$100\cdot 2^{24}+2^{25}=2^{25}(50+1)=51\cdot 2^{25}$

So $k=51$

21. Let circles $(x-0)^2+(y-4)^2=k$ and $(x-3)^2+(y-0)^2=1^2$ touches each other than find the maximum value of 'k'

Ans. 36.00

Sol. Two circles touches each other if $C_1C_2=|r_1\pm r_2|$

Distance between $C_2(3,0)$ and $C_1(0,4)$ is either $\sqrt{k}+1$ or $|\sqrt{k}-1|$ $\quad(C_1C_2=5)$

$\Rightarrow \sqrt{k}+1=5$ or $|\sqrt{k}-1|=5 \Rightarrow k=16$ or $k=36 \Rightarrow$ maximum value of k is 36.

22. Let $|\vec{a}|=\sqrt{3}, |\vec{b}|=5, \vec{b}\cdot\vec{c}=10$ angle between $\vec{b}$ & $\vec{c}$ equal to $\frac{\pi}{3}$

If $\vec{a}$is perpendicular to $\vec{b}\times\vec{c}$ then find the value of $|\vec{a}\times(\vec{b}\times\vec{c})|$

Ans. 30

Sol. $\vec{b}\cdot\vec{c}=10 \Rightarrow |\vec{b}||\vec{c}|\cos\left(\frac{\pi}{3}\right)=10 \Rightarrow 5\cdot|\vec{c}|\cdot\frac{1}{2}=10 \Rightarrow |\vec{c}|=4$

Also $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

$$|\vec{a}\times(\vec{b}\times\vec{c})=|\vec{a}||\vec{b}\times\vec{c}|\sin\left(\frac{\pi}{2}\right)$$

$$\sqrt{3}\times|\vec{b}||\vec{c}|\sin\frac{\pi}{3}\times 1=\sqrt{3}\times 5\times 4\times\frac{\sqrt{3}}{2}=30$$

23. Number of common terms in both sequence $3, 7, 11, \ldots 407$ and $2, 9, 16, \ldots 905$ is

Ans. 14.00

Sol. First common term$=23$

Common difference $=7\times 4=28$

Last term $\leq 407 \Rightarrow 23+(n-1)\times 28\leq 407$

$\Rightarrow (n-1)\times 28\leq 384 \Rightarrow n\leq 13\cdot 71+1$

$n\leq 14\cdot 71$

So $n=14$

24. If minimum value of term free from x for $\left(\frac{x}{\sin\theta}+\frac{1}{x\cos\theta}\right)^{16}$ is L_1 in $\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$ and L_2 in $\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right]$ find $\frac{L_2}{L_1}$

Ans. 16

Sol. $T_{r+1}={}^{16}C_r\left(\frac{x}{\sin\theta}\right)^{16-r}\left(\frac{1}{x\cos\theta}\right)^r$

For $r=8$ term is free from 'x'

$$T_9=\frac{{}^{16}C_8 1}{\sin^8\theta\cos^8\theta}$$

$$T_9={}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$$

$$\text{in }\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right], L_1={}^{16}C_8 2^8 \qquad \because\left\{\text{Min value of } L_1 \text{ at } \theta=\frac{\pi}{4}\right\}$$

$$\text{in }\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right], L_2={}^{16}C_8\frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8}={}^{16}C_8\cdot 2^8\cdot 2^4\{\because \text{min value of } L_2 \text{ at } \theta=\frac{\pi}{8}$$

$$\frac{L_2}{L_1}=\frac{{}^{16}C_8\cdot 2^8 2^4}{{}^{16}C_8\cdot 2^8}=16$$