13. A little of dry air at STP expands adiabatically to a volume of $3l$. If $\gamma = 1.40$, then work done by air is ($3^{1.4} = 4.6555$) [take air to be an ideal gas]:-

Sol. $W_{adiabatic} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

At STP, Pressure $= P_1 = 100000\ Pascal$

Volume $= V_1 = 1\ litre$ (As given in question)

So, $P_1 Y_1^{\gamma} = P_2 V_2^{\gamma}$

$\Rightarrow 1 \times (1)^{\gamma} = P_2 \times (3)^{1.4}$

$$P_2 = \frac{1}{4.65}\ atm$$

$$W_{\text{adiabatic}} = \left(\frac{1 \times 1 - \frac{1}{4.65} \times 3}{0.4} \right) \times 101.33$$

$= 90.5\ Joule$

14. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)T$, then what will be the expression for the electric field?

Sol. Given $\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)T$

$|\vec{E}| = BC = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\,\hat{j} \times (3 \times 10^8)$

$= 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)T$

Wave is propagating in x direction i.e., in i direction

The direction of EMW wave is in the direction of $\vec{E} \times \vec{B}$

Since B is in j direction and EMW is in i direction. Therefore E is in $(-k)$ direction.

15. A polarizer-analyzer set is adjusted such that the intensity of light coming out of the analyzer is just 10% of the original intensity. Assuming that the polarizer-analyzer set does not absorb any light, the angle by which the analyzer needs to be rotated further to reduce the output intensity to be zero is:

A) 45° B) 71.6° C) 90° D) 18.4°

Sol. D

Output intensity is given by $I = I_0 \cos^2(\theta)$

Initial output intensity = 10% of I_0

i.e., $\frac{10 I_0}{100} = I_0 \cos^2(\theta) \Rightarrow \theta = 71.57$

Final output intensity = 0

Means new angle is 90°

The angle by which the analyser needs to be rotated further is $90° - \theta = 18.4°$

16. Speed of transverse wave of a straight wire ($mass = 6.0\ g, length = 60\ cm$ and area of cross-section $= 1.0\ mm^2$) is $90\ ms^{-1}$. If the young's modulus of wire is $1.6 \times 10^{11} Nm^{-2}$, the extension of wire over its natural length is:

Sol. The linear mass density

$$\mu = \frac{m}{l} = \frac{6 \times 10^{-3}}{60 \times 10^{-2}}$$

$= 10^{-2} Kgm^{-1}$

Given $v = 90\ ms^{-1}$

$$v = \sqrt{\frac{T}{\mu}}$$

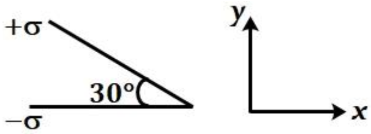
$T = \mu v^2 = 81\ N$

Youngs modulus

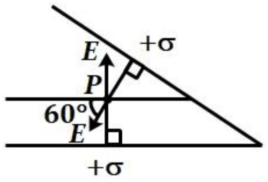
$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

$$\Delta l = \frac{FL}{AY} = \frac{81 \times 0.6}{10^{-6} \times 10 \times 10^{11}} = 0.03\ mm$$

17. Two infinite planes each with uniform surface charge density $+\sigma$ are kept in each such a way that the angle between them is 300. The electric field in the region shown between them is given by:



Sol. Assuming sheet to be non conducting



$$E_{net} = E\left(\left(1-\frac{\sqrt{3}}{2}\right)\hat{j} - \frac{1}{2}\,\hat{\imath}\right)$$

So, $E = \frac{\sigma}{2\varepsilon_0}$

So,$E_{net} = \frac{\sigma}{2\varepsilon_0}\left(\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\,\hat{x}\right)$

Where y and x are corresponding unit vectors.

18. If we used a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 m, the focal length of the eyepiece should be close to :

A) 12 mm B) 33 m C) 22 m D) 2 mm

Sol. C

Case-1: If final image is at least distance of clear vision

$$\text{M.P.} = \frac{L}{f_0}\left(1+\frac{d}{f_e}\right)$$

$$375 = \frac{150}{5}\left[1+\frac{25}{f_0}\right]$$

$$\frac{375}{30} = 1 + \frac{25}{f_e}$$

$$\frac{345}{30} = \frac{25}{f_0}$$

$$f_e = \frac{750}{345} = 2.17\ cm$$

$$f_e \approx 22\ mm$$

Case -2: If final image is at infinity

$$\text{M.P.} = \frac{L}{f_0}\left(\frac{D}{f_0}\right) = 375$$

$$f_e = 20\ mm$$

19. Visible height of wavelength $6000\times10^{-8} cm$ falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction min. is at 60 from the central max. If the first minimum is produced at θ, the θ is close to,

A) 25° B) 45° C) 20° D) 30°

Sol. A

$d\sin\theta$ = path difference $= 2\lambda$

So, $d\sin 60 = 2\lambda$

$$\Rightarrow \frac{\lambda}{d} = \frac{\sqrt{3}}{4}$$

For first minima,

$d\sin\theta_2 = \lambda$

$$\sin\theta_2 = \frac{\lambda}{d} = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right) = 25.64° = 25°$$

20. Consider a coil of wire carrying current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area given by ϕ_o. Which of the following is correct?

A) $\phi_i = -\phi_o$ B) $\phi_i > \phi_o$ C) $\phi_i < \phi_o$ D) $\phi_i = \phi_o$

Sol. A

Flux going right comes back to the left (forms closed loop)

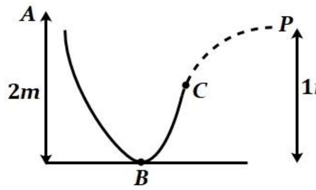
$\int B \cdot dA = 0$

Flux inside the coil come back through outside

$\phi_{\text{coil}} = -\phi_{\text{outside}}$

$\phi_0 = -\phi_1$

21. A particle ($m = 1\ kg$) slides down a frictionless block (AOC) starting from rest at a point ($height\ 2\ m$). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P ($height\ 1\ m$), the kinetic energy of the particle (in J) is (take $g = 10\ ms^{-2}$):-



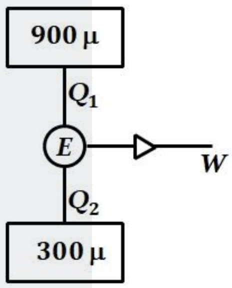
Sol.

Loss of potential energy = Gain in kinetic energy

$1 \times 10(2-1) = K_f - K_i = K_f - 0$

$K_f = 10\ J$

22. A carnot engine separates between two reservoirs of temperature $900\ K$ and $300\ K$. The engine performing $1200\ J$ of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir in a cycle is:-



Sol. Given, $W = 1200\ J$

$\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{900} = \frac{6}{9}$

$\eta_{carnot} = \frac{W}{Q_1}$

$So, \frac{W}{Q_1} = \frac{6}{9} \Rightarrow Q_1 = 1800\ J$

So, $W = Q_1 - Q_2 \Rightarrow Q_2 = Q_1 - w = 1800 - 1200 = 600\ J$

23. A loop $ABCDEFA$ of straight edges has six corner points $(0,0,0), B(5,0,0), C(5,5,0), D(0,5,0), E(0,5,5)$ and $F(0,0,5)$. The magnetic field in this region is $\vec{B} = (3\hat{\imath} + 4\hat{\jmath})T$. The quantity of flux through the loop $ABCDEFA$ (in Wb) is:-

Sol. $\vec{B} = (3\hat{\imath} + 4\hat{\jmath})T$

Total area vector = area of $ABCD$ + area of $DEFA = 5^2\hat{k} + 5^2\hat{\imath} = 25((\hat{\imath} + \hat{k})$

Total Magnetic flux = $\vec{B} \cdot \vec{A} = (3\hat{\imath} + 4\hat{\jmath}) \cdot \left(25(\hat{\imath} + \hat{k})\right) = (75 + 100)wb = 175wb$

24. A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5} W/cm^2$ is comprised of wavelength $\lambda = 310\ nm$. It falls normally on a metal (work function $\phi = 2\ eV$) of surface area $1\ cm^2$. If one in 10^3 photons ejects an electron, total number of electrons ejected in $1s$ is 10^x ($hc = 1240\ eVnm, 1\ eV = 1.6 \times 10^{-19} J$), then x is :-

Sol. Energy in $1\ second = 6.4\backslash timies\ 10^{-5} \times 1 \times 1 = 6.4 \times 10^{-5}\ Joule$

Energy $(eV) = \frac{6.4 \times 10^{-5}}{1.6 \times 10^{-19}} = 4 \times 10^{14} eV$

Energy of one photon $= \frac{12400}{3100} = 4\ eV$

Number of photon $= \frac{4 \times 10^{14}}{10^3} = 10^{14}$

Number of e; electrons $= \frac{10^{14}}{10^3} = 10^{11}$

So, $x = 11$

25. A non isotropic solid metal cube has coefficients of linear expansion as: $5 \times 10^{-5}/°C$ along the x-axis and $5 \times 10^{-6}/°C$ along the y-axis and z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}/°C$, then the value of C is:-

Sol. Since, it is cube, so all it's side is equal. Let the length of cube be L

Given, $\alpha_x = 5 \times 10^{-5}/°C$ and $\alpha_y = 5 \times 10^{-1}/°C = \alpha_z$

Let the coefficient of volume expansion be γ

After increase in temperature:-

Increase in volume of cube $= L^3 \times \gamma$ (1)

And we can also write it as:-$L(1 + \alpha_x)(1 + \alpha_y)(1 + \alpha_z)$ (2)

From equation 1 and 2 we get:-

$L^3 = L^3(1 + \alpha_x)(1 + \alpha_y)(1 + \alpha_z)$

$\Rightarrow Y = (\alpha_x + \alpha_y + \alpha_z)$ by igonoring $\alpha_x\alpha_y, \alpha_x\alpha_z; \alpha_y\alpha_z$, and $\alpha_x\alpha_y\alpha_z$

$\Rightarrow Y = (5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6})$

$\Rightarrow \gamma = 6 \times 10^{-5}/°C$

From question :-$\gamma = C \times 10^{-6}/°C$

So, $C = 60$