20. A solid sphere of density $\rho = \rho_0\left(1-\frac{r^2}{R^2}\right), 0 < r \le R$ just floats in a liquid then density of liquid is-(r is distance from centre of sphere)

A) $\frac{2}{5}\rho_0$ B) $\frac{5}{2}\rho_0$ C) $\frac{3}{5}\rho_0$ D) $\rho_0 s$

Ans. A

Sol. $\rho = \rho_0\left(1-\frac{r^2}{R^2}\right) \quad 0 < r \le R$

$mg = B$

$\int \rho(4\pi r^2 dt) = \rho_L \frac{4}{3}\pi R^3$

$\int \rho_0\left(1-\frac{r^2}{R^2}\right)4\pi r^2 dr = \rho_L \frac{4}{3}\pi R^3$

$\int_0^R \rho_0 4\pi\left(r^2 - \frac{r^2}{R^2}\right)dr$

$= \rho_{04\pi}\left(\frac{r^3}{3} - \frac{r^2}{5R^2}\right)_0^R \rho_L \frac{4}{3}\pi R^3$

$\frac{2}{5}\rho_0 = \rho_L$

21. Two masses each with mass 0.10 kg are moving with velocities 3 m/s along x-axis and 5 m/s along y-axis respectively. After an elastic collision one of the mass moves with a velocity $4\,\hat{\imath} + 4\hat{\jmath}$. The energy of other mass after collision is $\frac{x}{10}$ then x is.

Ans. 1

Sol. For elastic collision $KE_i = KE_f$

$\frac{1}{2}m \times 25 + \frac{1}{2} \times m \times 9$

$= \frac{1}{2}m \times 32 + \frac{1}{2}mv^2$

$34 = 32 + v^2$

$KE = \frac{1}{2} \times 0.1 \times 2 = 0.1\,J = \frac{1}{10}$

$x = 1$

22. A plano convex lens of radius of curvature 30 cm and refractive index 1.5 is kept in air. Find its focal length (in cm).

Ans. 60 cm

Sol. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$R_1 = \propto$

$R_2 = -30\ cm$

$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$

$\frac{1}{f} = \frac{0.5}{30}$

$f = 60\ cm$

23. Position of two particles A and B as a function of time are given by $X_A = -3t^2 + 8t + c$ and $Y_B = 10 - 8t^3$. The velocity of B with respect to A at $t = 1$ is v . Find v.

Ans. 580

Sol. $X_1 = -3t^2 + 8t + c$

$\vec{v}_A = (-6t + 8)\hat{\imath} = 2\hat{\imath}$

$Y_B = 10 - 8t^3$

$\vec{v}_B = -24t^2\hat{\jmath}$

$\sqrt{v} = |\vec{v}_B - \vec{v}_A|$

$= |-24\hat{\jmath} - 2\hat{\imath}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

$v = 580$

24. An open organ pipe of length $1m$ contains a gas whose density is twice the density of atmosphere at STP. Find the difference between fundamental and second harmonic frequencies if speed of sound in atmosphere is 300 m/s.

Ans. 105.75 Hz

Sol. $V = \sqrt{\frac{B}{\rho}}$

$$\frac{V_{\text{pipe}}}{V_{\text{air}}} = \frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$$

$$V_{\text{pipe}} = \frac{V_{\text{air}}}{\sqrt{2}}$$

$$f_n = \frac{(n+1)V_{\text{pipe}}}{2l}$$

$$f_1 - f_0 = \frac{V_{\text{pipe}}}{2l} = \frac{300}{2\sqrt{2}} = 105.75\ Hz\left(\text{if } \sqrt{2} = 1.41\right)$$

$$= 106.05\ Hz\ \left(\text{if } \sqrt{2} = 1.414\right)$$

25. Four resistors of 15Ω, 12Ω, 4Ω and 10Ω given in cyclic order to form a wheat stone bridge. What resistance (in Ω) should be connected in parallel across the 10Ω resistor to balance the wheat stone bridge.

Ans. 10

Sol. $\frac{10R}{10+R} \times 12 = 15 \times 4$

On solving

$R = 10\Omega$