Avanti

JEE Mains 2020

9th Jan Shift 1 Paper – Mathematics

1. Find the number of solution of $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|, x \in [0, 2\pi]$

A) 2 B) 4 C) 6 D) 8

Ans. D

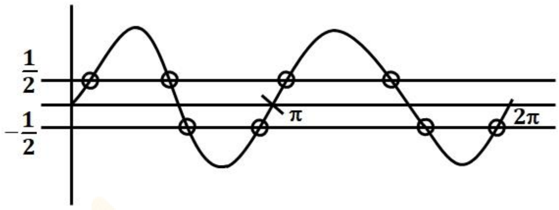
Sol. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$

$\log_{1/2}|\sin x \cos x| = 2$

$|\sin x \cos x| = \frac{1}{4}$

$\sin 2x = \pm\frac{1}{2}$

Number of solution $= 8$



2. If e_1 and e_2 are eccentricities of $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and $\frac{x^2}{9} - \frac{y^2}{4} = 1$, respectively and if the point (e_1, e_2) lies on ellipse $15x^2 + 3y^2 = k$. Then find value of k

A) 14 B) 15 C) 16 D) 17

Ans. C

Sol. $e_1 = \sqrt{1 - \frac{4}{18}} = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$

$e_2 = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$

$15e_1^2 + 3e_2^2 = k \Rightarrow k = 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right)$

$\therefore k = 16$

3. Find the integration $\int \frac{dx}{(x-3)^{6/7} \cdot (x+4)^{8/7}}$

A) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$ B) $7\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + c$ C) $7\left(\frac{x-3}{x-4}\right)^{\frac{6}{7}} + c$ D) $7\left(\frac{x+4}{x-3}\right)^{\frac{6}{7}} + c$

Ans. A

Sol. $\int \left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}} \frac{1}{(x+4)^2} dx$

Let $\frac{x-3}{x+4} = t^7, \frac{7}{(x+4)^2} dx = 7t^6 dt$

$\int t^{-6} t^6 dt = t + c$

4. If $\left|\frac{z-i}{z+2i}\right| = 1, |z| = \frac{5}{2}$ then value of $|z + 3i|$ is

A) $\frac{7}{2}$ B) $\sqrt{10}$ C) $\sqrt{5}$ D) $\sqrt{3}$

Ans. A

Sol. $x^2 + (y-1)^2 = x^2 + (y+2)^2$

$-2y + 1 = 4y + 4$

$6y = -3 \Rightarrow y = -\frac{1}{2}$

$x^2 + y^2 = \frac{25}{4} \Rightarrow x^2 = \frac{24}{4} = 6 \Rightarrow z = \pm\sqrt{6} - \frac{i}{2}$

$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$

$|z + 3i| = \frac{7}{2}$

5. $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \ldots \infty =$

A) $\sqrt{2}$ B) 2 C) $2^{\frac{1}{4}}$ D) 1

Ans. A

Sol. $2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+\ldots\infty} = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\infty} = \sqrt{2}$

6. Value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is

A) $\frac{1}{2\sqrt{2}}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{1}{2}$ D) $-\frac{1}{2}$

Ans. A

Sol. $\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]\left[\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right] - 3\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$

$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right] = \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$

7. Find the value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x}dx$

A) π^2 B) $2\pi^2$ C) $3\pi^2$ D) $4\pi^2$

Ans. A

Sol. $\int_0^{\pi} \frac{x\sin^8 x}{\sin^8 x+\cos^8 x} + \frac{(2\pi - x)\sin^8 x}{\sin^8 x+\cos^8 x}dx$

$$= \int_0^{\pi} \frac{2\pi\sin^8 x}{\sin^8 x + \cos^8 x}dx$$

$$= 2\pi\int_0^{\pi/2} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x}dx$$

$$= 2\pi\int_0^{\pi/2} 1\,dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

8. If $f(x) = a + bx + cx^2$ where $a, b, c \in R$ then $\int_0^1 f(x)\,dx$ is

A) $\frac{1}{3}\left(f(1) + f(0) + 2f\left(\frac{1}{2}\right)\right)$ B) $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$

C) $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$ D) $\frac{1}{6}\left(f(1) - f(0) + 4f\left(\frac{1}{2}\right)\right)$

Ans. B

Sol. $\int_0^1(a + bx + cx^2)\,dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}\Big|_0^1 = a + \frac{b}{2} + \frac{c}{3}$

$f(1) = a + b + c$

$f(0) = a$

$f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

Now $\frac{1}{6}\left(f(1) + f(0) + 4f\left(\frac{1}{2}\right)\right)$

$= \frac{1}{6}\left(a + b + c + a + 4\left(a + \frac{b}{2} + \frac{c}{4}\right)\right)$

$= \frac{1}{6}(6a + 3b + 2c) = a + \frac{b}{2} + \frac{c}{3}$

9. If number of 5 digit numbers which can be formed without repeating any digit while tenth place of all of the numbers must be 2 is $336\,k$ find value of k

A) 8 B) 7 C) 6 D) 5

Ans. A

Sol.

			2	

Number of numbers $= 8 \times 8 \times 7 \times 6 = 2688 = 336k \Rightarrow k = 8$

10. $A(3,-1), B(1,3), C(2,4)$ are vertices of ΔABC if D is centroid of ΔABC and P is point f intersection of lines $x+3y-1=0$ and $3x-y+1=0$ then which of the following points lies on line joining D and P

A) $(-9,-7)$ B) $(-9,-6)$ C) $(9,6)$ D) $(9,-6)$

Ans. B

Sol. $D(2,2)$

Point of intersection $P\left(-\frac{1}{5},\frac{2}{5}\right)$

equation of line DP

$8x-11y+6=0$

11. If $f(x)$ is twice differentiable and continuous function in $x \in [a,b]$ also $f'(x)>0$ and $f''(x)<0$ and $c \in (a,b)$ then $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

A) $\frac{b-c}{c-a}$ B) 1 C) $\frac{a+b}{b-c}$ D) $\frac{c-a}{b-c}$

Ans. D

Sol. Lets use LMVT for $x \in [a,c]$

$$\frac{f(c)-f(a)}{c-a} = f'(\alpha), \alpha \in (a,c)$$

also use LMVT for $x \in [c,b]$

$$\frac{f(b)-f(c)}{b-c} = f'(\beta), \beta \in (c,b)$$

$\because f''(x)<0$

$\Rightarrow f'(x)$ is decreasing

$f'(\alpha) > f'(\beta)$

$$\frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$$\frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c} \quad (\because f(x) \text{ is increasing})$$

12. If plane

$x+4y-2z=1$

$x+7y-5z=\beta$

$x+5y+\alpha z=5$

intersection in a line $(R \times R \times R)$ then $\alpha+\beta$ is equal to

A) 0 B) 10 C) -10 D) 2

Ans. B

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$

$(7\alpha+25)-(4\alpha+10)+(-20+14)=0$

$3\alpha+9=0$

$\Rightarrow \alpha=-3$

Also $D_z=0$

$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$

$1(35-5\beta)-(15)+1(4\beta-7)=0$

$\beta=13$

13. For observation x_i given $\sum_{i=1}^{10}(x_i-5)=10$ and $\sum_{i=1}^{10}(x_i-5)^2=40$. If mean and variance of observations $(x_1-3),(x_2-3)\ldots(x_{10}-3)$ is λ & μ respectively then ordered pair (λ,μ) is

A) $(3,3)$ B) $(1,3)$ C) $(3,1)$ D) $(1,1)$

Ans. A

Sol. Mean (x_i-5)

$$=\frac{\sum(x_i-5)}{10}=1$$

$\therefore \lambda = \{\text{Mean } (x_i-5)\}+2=3$

$$\mu = \text{var } (x_i-5) = \frac{\sum(x_i-5)^2}{10} - \frac{\sum(x_i-5)}{10} = 3$$

14. In a bag there are 20 cards 10 names A and another 10 names B. Cards are drawn randomly one by one with replacement then find probability that second A comes before third B.

A) $\frac{13}{16}$ B) $\frac{11}{16}$ C) $\frac{7}{16}$ D) $\frac{9}{16}$

Ans. B

Sol. $AA + ABA + BAA + ABBA + BBAA + BABA = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

15. The negation of '$\sqrt{5}$ is an integer of 5 is an irrational number' is

A) $\sqrt{5}$ is an integer and 5 is not an irrational number

B) $\sqrt{5}$ is not an integer and 5 is an irrational number

C) $\sqrt{5}$ is not an integer or 5 is not an irrational number

D) $\sqrt{5}$ is not an integer and 5 is not an irrational nunmber

Ans. D

Sol. $\sqrt{5}$ is not an integer and 5 is not an irrational number $\sim (p \vee q) = \sim p \wedge \sim q$

16. If a circle touches y-axis at $(0, 4)$ & passes through $(2, 0)$ then which of the following can not be the tangent to the circle

A) $3x + 4y - 6 = 0$ B) $3x + 4y - 24 = 0$

C) $4x - 3y - 17 = 0$ D) $4x + 3y - 6 = 0$

Ans. A

Sol. Equation of family of circle

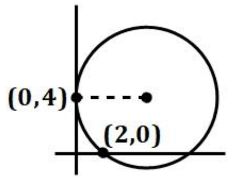
$(x - 0)^2 + (y - 4)^2 + \lambda x = 0$

$\Rightarrow$ passes $(2, 0)$

$4 + 16 + 2\lambda = 0 \Rightarrow \lambda = -10$

$x^2 + y^2 - 10x - 8y + 16 = 0$

Centre $(5, 4)$. $R = \sqrt{25 + 16 - 16} = 5$



17. If $f'(x) = \tan^{-1}(\sec x + \tan x), x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $f(0) = 0$ then the value of $f(1)$ is

A) $\frac{\pi+1}{4}$ B) $\frac{\pi-1}{4}$ C) $\frac{\pi+1}{2}$ D) 0

Ans. A

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right) = \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$

$(f'(x))dx = \frac{\pi}{4} + \frac{x}{2}dx$

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$

$f(0) = c = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

So $f(1) = \frac{\pi+1}{4}$

18. A sphere of 10 cm radius has a uniform thickness of ice around it. Ice is melting at rate 50 cm^3/min when thickness is 5 cm then rate of change of thickness

A) $\frac{1}{36\pi}$ B) $\frac{1}{18\pi}$ C) $\frac{1}{9\pi}$ D) $\frac{1}{12\pi}$

Ans. B

Sol. Let thickness $= x\ cm$

Total volume $v = \frac{4}{3}\pi(10 + x)^3$

$\frac{dv}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$ (i)

Given $\frac{dv}{dt} = 50\ cm^3/min$

At $x = 5\ cm$

$50 = 4\pi(10 + 5)^2 \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{18\pi} cm/min$

19. Find number of real roots of equation $e^{4x}+e^{3x}-4e^{2x}+e^{x}+1=0$ is

A) 1 B) 2 C) 3 D) 4

Ans. A

Sol. Let $e^x = t \in (0,\infty)$

Given equation $t^4+t^3-4t^2+t+1=0$

$$t^2+t-4+\frac{1}{t}+\frac{1}{t^2}=0$$

$$\left(t^2+\frac{1}{t^2}\right)+\left(t+\frac{1}{t}\right)-4=0$$

Let $t+\frac{1}{t}=\alpha$

$(\alpha^2-2)+\alpha-4=0$

$\alpha^2+\alpha-6=0$

$\alpha=-3,2$

$\Rightarrow \alpha=2 \Rightarrow e^x+e^{-x}=2$

$x=0$ only solution

20. If $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, $B=adj\,(A)$ and $C=3A$ then $\frac{|adj\,B|}{|C|}$ is

A) 8 B) 4 C) 2 D) 16

Ans. A

Sol. $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}=((9+4)-1(3-4)+2(-1-3))=13+1-8=6$

$|adj\,B|=|adj\,adj\,A|=|A|^{(n-1)^2}=|A|^4=(36)^2$

$|C|=|BA|=3^3\times 6$

$\frac{|adj\,B|}{|C|}=\frac{36\times 36}{3^3\times 6}=8$

21. $(1+x)\frac{dy}{dx}=\left((1+x)^2+(y-3)\right)$, If $y(2)=0$ then $y(3)=?$

Ans. 3

Sol. $\frac{dy}{dx}=(1+x)+\left(\frac{y-3}{1+x}\right)$

$$\frac{dy}{dx}-\frac{1}{(1+x)}y=(1+x)-\frac{3}{(1+x)}$$

I.F. $=e^{-\int\frac{1}{(1+x)}dx}=\frac{1}{(1+x)}$

$$\therefore \frac{d}{dx}\left(\frac{y}{1+x}\right)=1-\frac{3}{(1+x)^2}$$

$$\frac{y}{1+x}=x+3(1+x)^{-1}+c$$

$$y=(1+x)\left[x+\frac{3}{(1+x)}+c\right]$$

$\therefore$ at $x=2, y=0$

$\therefore 0=3(2+1+c)\Rightarrow c=-3$

$\therefore$ at $x=3, y=3$

22. $f(x)=\begin{cases}\frac{\sin(a+2)x+\sin x}{x}; & x<0\\ b; & x=0\\ \frac{(x+3x^2)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}}; & x>0\end{cases}$

Function is continuous at $x=0$, find $a+2b$.

Ans. 0

Sol. LHL $=a+3$

$f(0)=b$

RHL $=\lim_{h\to 0}\left(\frac{(1+3h)^{\frac{1}{3}}-1}{h}\right)=1$

$\therefore a=-2$

$b=1$

$\therefore a+2b=0$

23. Find the coefficient of x^4 in $(1 + x + x^2)^{10}$

Ans. 615

Sol. General term $\frac{10!}{\alpha!\beta!\gamma!}x^{\beta+2\gamma}$

for coefficient of $x^4 \Rightarrow \beta + 2\gamma = 4$

$\gamma = 0, \beta = 4, \alpha = 6$

$\Rightarrow \dfrac{10!}{6!\,4!\,0!} = 210$

$\gamma = 1, \beta = 2, \alpha = 7$

$\Rightarrow \dfrac{10!}{7!\,2!\,1!} = 360$

$\gamma = 2, \beta = 0, \alpha = 8$

$\Rightarrow \dfrac{10!}{8!\,0!\,2!} = 45$

Total = 615

24. If $\vec{P} = (a+1)\hat{\imath} + a\hat{\jmath} + a\hat{k}$

$\vec{Q} = a\hat{\imath} + (a+1)\hat{\jmath} + a\hat{k}$

$\vec{R} = a\hat{\imath} + a\hat{\jmath} + (a+1)\hat{k}$

and $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors and $3(\vec{P}\cdot\vec{Q})^2 - \lambda|\vec{R}\times\vec{Q}|^2 = 0$ then value of λ is

Ans. 1

Sol. $\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0 \Rightarrow a + 1 + a + a = 0 \Rightarrow a = -\frac{1}{3}$

$\vec{P} = \dfrac{2}{3}\hat{\imath} - \dfrac{1}{3}\hat{\jmath} - \dfrac{1}{3}\hat{k}$

$\vec{Q} = \dfrac{1}{3}(-\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$\vec{R} = \dfrac{1}{3}(-\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\vec{P}\cdot\vec{Q} = \dfrac{1}{9}(-2 - 2 + 1) = -\dfrac{1}{3}$

$\vec{R}\times\vec{Q} = \dfrac{1}{9}\begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix} = \dfrac{1}{9}\big(i(4-1) - j(-2-1) + k(1+2)\big)$

$= \dfrac{1}{9}(3i + 3j + 3k) = \dfrac{i + j + k}{3}$

$|\vec{R}\times\vec{Q}| = \dfrac{1}{3}\sqrt{3} \Rightarrow |\vec{R}\times\vec{Q}|^2 = \dfrac{1}{3}$

$3(\vec{P}\cdot\vec{Q})^2 - \lambda|\vec{R}\times\vec{Q}|^2 = 0$

$3\cdot\dfrac{1}{9} - \lambda\cdot\dfrac{1}{3} = 0 \Rightarrow \lambda = 1$

25. If points $A(2,4,0), B(3,1,8), C(3,1,-3), D(7,-3,4)$ are four points then projection of line segment AB on line CD.

Ans. 8

Sol. $\overrightarrow{AB} = (\hat{\imath}) - (3\hat{\jmath}) + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

$(\overrightarrow{AB}\cdot\overrightarrow{CD}) = \dfrac{4 + 12 + 56}{\sqrt{16 + 16 + 49}} = \dfrac{72}{9} = 8$